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Research Article

An Efficient Economic-Statistical Design of Simple Linear Profiles Using a Hybrid Approach of Data Envelopment Analysis, Taguchi Loss Function, and MOPSO

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Abstract

Statistically constrained economic design for profiles usually refers to the selection of some parameters such as the sample size, sampling interval, smoothing constant, and control limit for minimizing the total implementation cost while the designed profiles demonstrate a proper statistical performance. In this paper, the Lorenzen-Vance function is first used to model the implementation costs. Then, this function is extended by the Taguchi loss function to involve intangible costs. Next, a multi-objective particle swarm optimization (MOPSO) method is employed to optimize the extended model. The parameters of the MOPSO are tuned using response surface methodology (RSM). In addition, data envelopment analysis (DEA) is employed to find efficient solutions among all near-optimum solutions found by MOPSO. Finally, a sensitivity analysis based on the principal parameters of the cost function is applied to evaluate the impacts of changes on the main parameters. The results show that the proposed model is robust on some parameters such as the cost of detecting and repairing an assignable cause, variable cost of sampling, and fixed cost of sampling.

***Keywords*:** MOPSO; Economic-statistical design; Linear profiles; Quadratic loss function; Data envelopment analysis (DEA); Response Surface Methodology (RSM)

1. Introduction

Nowadays control charts are employed to monitor critical parameters of a process based on their probability distributions (Montgomery, 2005). Generally, most processes could not be executed in the state of in-control indefinitely. Thus, the continuous application of a control chart will identify the assignable cause. Also, the statistically constrained design for a control chart refers to determining its control limit(s) so as the chart exhibits good statistical performances in the in-control process conditions and the out-of-control state as well.
Designing control chart by considering the cost of application (called economic design) was firstly proposed by Duncan (1971) to minimize a cost objective function with only a single assignable cause. Lorenzen and Vance (1986) developed a model for the costs of implementation in many types of control charts. As the economical design usually has a poor statistical performance, Saniga (1989) applied statistical constraints to construct an economic-statistical model for designing a chart that takes into account both Type-I and Type-II errors.

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functional relationship called profiles. Walker and Wright (2002) introduced an example for the application of a profile. Mestek et al. (1994) used a similar idea to investigate a calibration process.
In this paper, a statistically constrained economic model is presented for linear profiles. In this model, the general cost function (named Lorenzen-Vance) is extended by the Taguchi loss function. In order to achieve an efficient design the data envelopment analysis (DEA) approach is employed. Due to the complexity involved, a meta-heuristic algorithm is utilized to solve the problem where its parameters are tuned using response surface methodology (RSM). In final section, a sensitivity analysis for the main parameters of the chart is performed to investigate their impacts on the efficiency of the designed monitoring method. In the next section, we will review the relevant literature.

2. Literature Review

The economic design of control charts for the first time is presented by Duncan (1971) when he proposed the design to select the parameters of the X-bar chart. Later, Duncan (1971) presented another model to be used in situations with multi-assignable causes. Saniga (1989) introduced a statistically constrained economic model (called economic-statistical) by adding two constraints (Type-I and Type-II errors) on Duncan's model. Elsayed and Chen (1994) developed an economic design for X-bar charts based on quadratic loss function. Costa and Rahim (2001) employing the Markov chain approach developed an economic model for X-bar charts with variable parameters. Chou et al. (2002) proposed the statistically constrained economic design for multivariate control charts by applying quality loss function. Chen & Yang (2002) introduced an economic design for the C-bar control chart with multi-assignable causes. Serel and Moskowitz (2006) used the Taguchi loss function for improving the Lorenzen-Vance cost model and monitoring the mean and the variance of a process simultaneously. Yang et al. (2012) applied a meta-heuristic algorithm to find the optimal design of X-bar and S control chart in a multi-objective environment. Control charts along with other tools are appropriate methods for implementing root cause analysis (RCA) in industries (Ershadi et al., 2018). Saghaei et al. (2014) proposed an economical design for EWMA chart using a genetic algorithm.

In contrast with economic and statistically constrained economic designs of classical control charts, there are a few works in the literature on the design of linear profiles. Noorossana et al. (2014) proposed both economic and statistically constrained economic designs of simple linear profiles. Ershadi et al (2015) developed an economic design model for a simple linear profile with variable sampling interval in Phase II. Ershadi et al. (2016) worked on design of simple linear profiles in adaptive environment.

In order to summarize the works reviewed on the economic-statistical designs, they are first categorized in four main groups. The first group involves published papers in the recent two decades that focus on the single-objective design of control charts considering an economic model. The works such as Barzinpour et al. (2013), Saniga (1989), Chou et al. (2008), Niaki et al. (2011), Saghaei et al. (2014), and Niaki and Ershadi (2012) are placed in this group. The second group is concerned with the multi-objective economic/statistical designs of control charts that start in 2012. The papers by Yang et al. (2011), Safaei et al. (2012), and Tavana et al. (2016) are among these works. The third group contains studies such as Noorossana et al. (2014) that were conducted on developing a single-objective optimization model for linear profiles. The fourth group is devoted to the studies focused on the multi-objective designs of linear profiles, in which no works have been conducted so far in the literature and the current research falls within it. In other words, the current research is performed with the aim of presenting a multiple objective optimization model for designing of linear profiles. Table 1 presents these groups and shows the research gaps.

Based on the above review, while many works are conducted on the statistically constrained economic design of charts, a model with multiple objectives is proposed for the first time in this paper to obtain the parameters of a simple linear profile. In addition to the above-mentioned contribution, the Lorenzen-Vance function is extended based on the Taguchi's loss function to incorporate all costs of implementing linear profiles described in Section 4.

In this paper, a new model is developed to improve the efficiency of an economic-statistical design of simple linear profiles based on the DEA approach that incorporates hidden implementation costs identified through the Taguchi loss function. The proposed model is solved by a combination of MOPSO and RSM. In what comes in the next section, some necessary backgrounds on simple linear profiles, cost functions, and DEA are provided.

Table1
Relevant studies in the design of control charts and profiles using meta-heuristics

Previous Studies	Control charts		Linear Profiles	
	Single objective	**Multi-Objective**	**Single objective**	**Multi-objective**
Barzinpour et al. (2013)	PSO			
Saniga (1989)	GA			
Chou & Cheng (2008)	GA			
Niaki et al. (2011)	PSO			
Saghaei et al. (2014)	GA			
Niaki & Ershadi (2012)	GA			
Niaki & Ershadi (2012)	ACO			
Safaei et al (2012)		NSGA-II		
Yang et al (2012)		MOPSO		
Tavana et al. (2016)		NSGA-III /MOPSO		
Noorossana et al. (2014)			GA	
Current research				DEA/MOPSO

3. Background

As stated above, this section provides some required background on simple linear profiles, the Lorenzen-Vance implementation cost functions, and data envelopment analysis.

3.1. Simple linear profile

Suppose the random variable Y is the output of a process having a linear relationship with an independent variable X as

$$Y = A_0 + A_1 X + \varepsilon_0 \quad ; \quad X_l \leq X \leq X_h, \tag{1}$$

where A_0 and A_1 are, respectively, the intercept and slope parameters and X_l and X_h define the range of X. The relationship defined in (1) is called a simple linear profile (Keramatpour et al., 2014). In Eq. (1), the ε_0s are normally and independently distributed variables with mean 0 and variance σ^2. To detect any change in the standard deviation or the average of the process, a sample j with n set points $x_1, x_2, \ldots, x_n$ selected in the range $[X_l, X_h]$ is first taken from the process to observe $y_{1j}, y_{2j}, \ldots, y_{nj}$. Then, assuming a linear relationship among the points in sample , $(x_1, y_{1j}), (x_2, y_{2j}), \ldots, (x_n, y_{nj})$, the least squares estimates for parameters A_0 and A_1 are:

$$a_{1j} = \frac{S_{xy(j)}}{S_{xx}} \quad ; \quad a_{0j} = \bar{Y} - a_{1j}\bar{X}, \tag{2}$$

where $\bar{X}$ and $\bar{Y}$ are the sample means of X and Y and

$$S_{xx} = \sum_{i=1}^{n} (x_i - \bar{x})^2 \quad ; \quad S_{xy} = \sum_{i=1}^{n} y_{ij}(x_i - \bar{x}). \tag{3}$$

Based on equations (2) and (3), the sample statistics a_{0j} and a_{1j} have the means A_0 and A_1 and normally distributed with the variances

$$\sigma_0^2 = \sigma^2(n^{-1} + \bar{x}^2)\sigma_0^2 = \sigma^2(n^{-1} + \bar{x}^2) \quad ; \quad \sigma_1^2 = \sigma^2 S_{xx}^{-1}. \tag{4}$$

Assuming the predicted value of Y as $\bar{Y} = a_{0j} + a_{1j}X$, the residual e_{ij} is the deviation of the observed and predicted values as

$$e_{ij} = y_{ij} - a_{0j} - a_{1j}x_i. \tag{5}$$

The independent random variables e_{ij}s are normally distributed with mean 0 and variance σ^2 estimated by

$$MSE_j = (n-2)^{-1} \sum_{i=1}^{n} e_{ij}^2. \tag{6}$$

For an in-control process mean, the residuals must be in control. One way to check is the use of the EWMA chart on the average residuals which is proposed by Kang and Albin (2000).

$$\bar{e}_j = \frac{\sum_{i=1}^{n} e_{ij}}{n} \tag{7}$$

In this case, the j^{th} sample statistic is the weighted average of the j^{th} average residual and the previous average residual defined in Eq. (8).

$$z_j = r\bar{e}_j + (1-r)z_{j-1} \tag{8}$$

In Eq. (8), $0 < r < 10$ is the weighting constant and $z_0 = 0$. However, in some cases, the average of the initial data $\bar{x}$ is used as the starting point, i.e. $z_0 = \bar{x}$. Furthermore, the upper control limit (UCL) and the lower control limit (LCL) of the EWMA chart are

$$UCL = L\sigma\sqrt{\frac{r}{(2-r)n}} \quad ; \quad LCL = -L\sigma\sqrt{\frac{r}{(2-r)n}} \tag{9}$$

An out-of-control signal is prompted when z_j is either less than LCL or it is greater than UCL.

To monitor the variance of the process, the range (R) chart is constructed using the control limits defined in Eq. (10).

$$UCL = \sigma(d_2 + Ld_3) \quad ; \quad LCL = \sigma(d_2 - Ld_3) \tag{10}$$

The coefficients d_2 and d_3 in Eq. (10) are proportional to n and are determined by corresponding tables (Montgomery, 2005). Here, the parameter L is related to the sensitivity of the EWMA chart and is determined by the economic-statistical design, developed later in Section 4.

The simulation method utilized in Kang and Albin (2000) is employed in this paper, where the above derivations are used to compute the average run lengths (ARL's). In the following sub-section, the Lorenzen-Vance cost function is introduced to determine the cost of implementing linear profiles.

3.2. The Lorenzen-Vance cost function

Lorenzen & Vance (1986) proposed a general model for the implementation costs of different types of control charts. Since then many researchers used this model for the economic design of their control charts. For example Molnau et al. (2001) presented an model for designing a MEWMA chart and also Niaki and Ershadi (2012) developed a model to design a MEWMA chart in which the Markov chain approach was employed for ARL calculations. As the Lorenzen-Vance model has been the only cost function used to design EWMA charts and noting its flexibility in the economic-statistical design of control charts, it is selected in this paper for the economic-statistical design of simple linear profiles.

The expected implementation cost according to the Lorenzen-Vance function is as follows.

$$C(n,h,L,r) = \frac{\left\{\frac{C_0}{\theta} + C_1(-\tau + nE + h(ARL_1) + \gamma_1 T_1 + \gamma_2 T_2) + \frac{SF}{ARL_0} + W\right\}}{\{\frac{1}{\theta} + \frac{(1-\gamma_1)ST_0}{ARL_0} - \tau + nE + h(ARL_1) + T_1 + T_2\}} + \frac{\left\{\left[\frac{a+bn}{h}\right]\left[\frac{1}{\theta} - \tau + nE + h(ARL_1) + \gamma_1 T_1 + \gamma_2 T_2\right]\right\}}{\{\frac{1}{\theta} + \frac{(1-\gamma_1)ST_0}{ARL_0} - \tau + nE + h(ARL_1) + T_1 + T_2\}} \qquad (11)$$

where the parameters are:
C_0 is the average nonconformities cost per hour while the process is on in-control state.
C_1 is the average nonconformities cost per hour while the process is on out-of-control state.
τ is the average duration between the time an assignable cause occurs and the last previous sample point. It is obtained as

$$\tau = \frac{\int_{jh}^{(j+1)h} e^{-\theta h}(t-jh)dt}{\int_{jh}^{(j+1)h} e^{-\theta h}dt} = \frac{1-(1+\theta h)e^{-\theta h}}{\theta(1-e^{-\theta h})} \qquad (12)$$

E is the required time for sampling and depicting an item.
ARL_0 is the average run length while the process is on in-control state.
ARL_1 is the average run length when the process is on out-of-control state.
T_0 is the required time to search for the cause of assignable condition when the chart signals a false alarm.
T_1 is the mean time to identify an assignable cause
T_2 is the mean time to correct and modify the process

$\gamma_1 = 0$ in situation the process is stopped while identifying assignable cause and is equal to 1 if it is progressed during the search.

$\gamma_2 = 0$ in situation the process is stopped while improving and is set equal to 1 if it is progressed while correcting or repairing.

S is the average number of samples when the process is on in-control state. It is calculated by

$$S = \frac{\exp(-\theta h)}{[1-\exp(-\theta h)]} \qquad (13)$$

F is the established cost of a false alarm.
W is the identifying and modifying cost for an assignable cause
a is the fixed sampling cost of each item.
b is the variable sampling cost of each item.

As some hidden implementation costs in the above function may not be properly considered, we will estimate it with the aid of the Taguchi loss function introduced in Section 4.

3.3. Data envelopment analysis

DEA is a methodology for evaluating multiple decision-making units (DMUs) in efficiency perspective when the production process establishes an arrangement of multiple outputs and inputs (Azizi and Kazemi Matin, 2018). The efficiency indicator which is generally used by DEA is as follows

$$\mathrm{E_r(s)} = \frac{\text{the weighted sum of outputs}}{\text{the weighted sum of inputs}} = \frac{\sum_{r=1}^{s} u_r(s) y_{r0}}{\sum_{i=1}^{m} v_i(s) x_{i0}} \qquad (14)$$

where
$\mathrm{E_r(s)}$ is the efficiency measure of DMUs in design s.
$u_r(s)$ is the r^{th} value of output y in design s;
$v_i(s)$ is the r^{th} value of input x in design s;
y_{r0} is the most affirmative weights determined to design r for output y;
x_{i0} is the most affirmative weights determined to design r for input x.

In the stage of calculating the relative efficiency, it is required to determine evaluation method of weights (Yaghoubi et al., 2016). There should be a common set of weights for all decision-making units. In the real world, however, determining these weights for each DMU is a difficult task. Therefore, DEA proposes a solution to this problem based on the efficiency measure.

Charnes et al. (1978) proposed the first DEA model called CCR that efficiency is obtained by dividing the weight composition of the outputs into the weight composition of the inputs in the fractional planning model as follows.

$$Max \sum_{r=1}^{s} u_r y_{r0} \Big/ \sum_{i=1}^{m} v_i x_{i0} \quad s.t. \quad \sum_{r=1}^{s} u_r y_{rj} \Big/ \sum_{i=1}^{m} v_i x_{ij} \le 1 \; ; \; j = 1,2,\dots,n \qquad (15)$$

$u_r \geq 0 \;\; ; \;\; v_i \geq 0$

Charnes & Cooper (1988) converted the CCR fractional programming model to CCR linear programming model by applying the limitation $\sum_{i=1}^{m} v_i x_{i0} = 1$. In this model, the efficiency of each unit is assumed to be constant on a scale. The CCR linear programming model is as follows.

$$\begin{aligned} & Max \sum_{r=1}^{s} u_r y_{r0} \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \;\; ; \;\; j = 1,2,\dots,n \\ & s.t. \\ & \sum_{i=1}^{m} v_i x_{i0} = 1 \\ & u_r \geq 0 \;\; ; \;\; v_i \geq 0 \end{aligned} \tag{16}$$

In the next section, an extension of the cost model is first given for a statistically constrained economic design of simple linear profiles using the Taguchi's loss function.

4. An Extension of the Lorenzen-Vance Function

As stated in the previous section, the presented cost function is based on the internal cost of control charts or profiles and do not include the external cost of implementation. Customer needs and requirements have main impact on the forced costs in any organization and appropriately need to be considered in organizational accounting (Ershadi and Omidvar, 2018). Taguchi and Wu (1979) developed the quality of an item as a loss occurred since the time it is introduced to the market, based on which the average total loss is obtained as an indicator of the performance. Safaei et al. (2012) presented a multi-objective statistically constrained economic model for Shewhart control charts by considering the Taguchi's loss function. Serel (2009) used this function to estimate two parameters C_0 and C_1 and showed that if T is the target value of any quality characteristic for the monitored and K is the coefficient in Taguchi loss when the process is in control, then T is equal to mean of the quality characteristic(μ_0). Thus, the average quality cost per unit of the product when the process is on in-control state (J_0) is as follows

$$J_0 = \mathrm{K}[\sigma_0^2 + (\mu_0 - T)^2]. \tag{17}$$

However, when the process is in out-of-control state, the process mean will change from μ_0 to μ_1 and the expected cost per each item (J_1) becomes

$$J_1 = K[\rho^2 \sigma_0^2 + (\mu_0 - T)^2 + \delta^2 \sigma_0^2 - 2\delta\sigma_0(\mu_0 - T)] \tag{18}$$

In Eq. (18), ρ is the ratio of the standard deviation in out-of-control state to standard deviation on in-control state, which is calculated as follows.

$$\rho = \frac{\sigma_1}{\sigma_0} \tag{19}$$

In addition, the Taguchi loss coefficient K is a fixed number that depends on the cost of rework, waste, and the size of tolerance characteristic. It can be estimated by

$$K = \frac{A}{(x - T)^2} \tag{20}$$

Assuming P as the units which are produced per hour, the average loss per hour when the process is on in-control state is calculated as follows.

$$C_0 = J_0 P \tag{21}$$

In addition, the average loss per hour when the process is on out-of-control state is obtained by

$$C_1 = J_1 P \tag{22}$$

Replacing the parameters C_0 and C_1 in the presented cost function, the average total loss (ATL) is calculated. The next sub-section provides the total model of this study.

4.1. An efficient economic-statistical model

Based on the backgrounds provided in Section 3 and the proposed extended cost function in Section 4, an efficient statistically constrained economic model to design profiles is formulated as follows.

$$\begin{aligned} & Min\ ATL(n,h,r,l) \\ & Min\ ARL_1 \\ & Max\ ARL_0 \\ & Min\ ATS_1 \\ & Max\ ATS_0 \\ & s.t. \\ & \quad ARL_0 > ARL_L \\ & \quad ARL_1 < ARL_U \\ & \quad 0 < r < 1 \\ & \quad h > 0;\ l > 0 \;\; ; \;\; n \text{ an integer} \end{aligned} \tag{23}$$

The variables used in (23) are defined as shown below.
n : is the number of the set points;
h : is the sampling interval;
l: is the rate to assess the average run lengths;
r : is the weighting variable in the applied EWMA-R chart.
ARL_L: is the lower limit on the ARL when the process is in-control;
ARL_U: is the upper limit on the ARL when the process is out-of-control;

ATS_0: is the average time to signal when the process is in control;
ATS_1: is the average time to signal when the process is out of control;
$ATL(n, h, r, l)$: is the average total loss.

Figure 1 shows a general flowchart to optimize and validate Model (23).

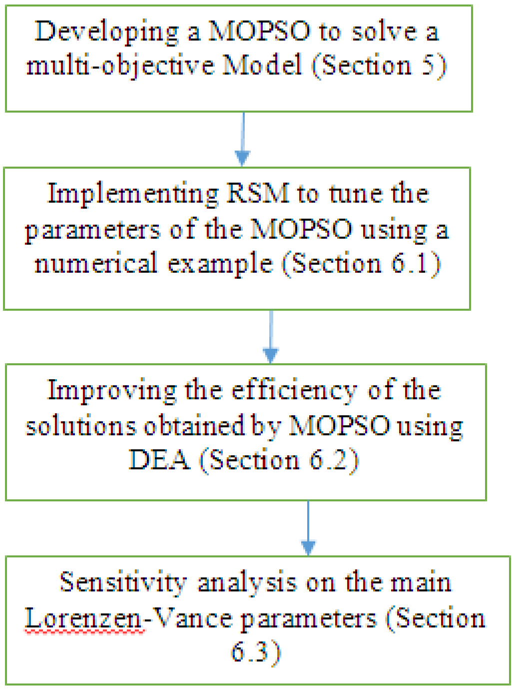

Fig.1. The general structure to optimize Model (23)

In the next section, by employing a meta-heuristic algorithm near-optimum solution of the problem at hand is found and to aid finding an efficient design of simple linear profiles.

5. A Meta-Heuristic Solution Algorithm

There are several meta-heuristic algorithms to solve the extended Lorenzen and Vance function introduced in Section 4. The goal of these algorithms is to efficiently investigate feasible region to find optimum results (Sadigh et al., 2010). Liu et al. (2017) optimized the parameters of a modified MEWMA chart using a PSO algorithm, presented by Kennedy and Eberhart (1997). Due to the ability, flexibility, and high speed of these algorithms, they have been utilized in many studies to solve various types of complex optimization problems. Niaki et al. (2011) compared the performances of four meta-heuristic algorithms when they were used for a statistically constrained economic design of charts and concluded that PSO is the best for solving their problem. As such, a MOPSO is utilized in the next sub-section in order to solve the complex optimization problem at hand.

Multi-objective particle swarm optimization (MOPSO)

In PSO, each solution (named particle) x_n is assumed as a member in the swarm of N particles are established with a velocity vector which specifies its location at the consecutive time step. The velocities for each particle are revised to fly towards two different paths: their personal best, P_n, to benefit and select the best results found until now, and the global best, G, which is the best solution obtained until now by the total swarm. A typical multi-objective PSO (MOPSO) involves several objectives to be optimized simultaneously. Coello et al. (2004) proposed this algorithm and Tavana et al. (2016) used MOPSO and non-dominated sorting genetic algorithm II (NSGA-II) to solve involved in the statistical constrained economic design of control charts.
The steps of the MOPSO algorithm are summarized as follows:

1. Set starting value for the population Pop
2. Set starting value for the speed of each particle
3. Appraise each particle in Pop
4. Save the locations of the particles that show non-dominated vectors in Rep
5. Generate hypercubes of the search space explored so far
6. Set starting value for the memory of each particle
7. Do the following steps until the maximum number of cycles is reached:
    a. Compute the speed of each particle
    b. Compute the new positions adding the seed obtained by the previous step as

$$Pop[i] = Pop[i] + Vel[i] \quad (24)$$

    c. Maintain the particles within the search space in case they go beyond their boundaries.
    d. Evaluate each of the particles
    e. Update the contents with the geographical representation of the particles
    f. When the current position of the particle is better than the position contained in its memory, the particle's position is updated using

$$P_{best}[i] = Pop[i] \quad (25)$$

    g. Increment the loop counter
8. End While

In the next section, a numerical example is provided to solve the statistically constrained economic design problem at hand.

6. A Numerical Example from a Real-World Application

In this section, the proposed model is applied to a case which is studied by Kang and Albin (2016). They inspired their example by the first principles in physics. The semiconductor manufacturing problem that they addressed takes place during the etch step. If a mass flow controller (MFC) is in-control then the measured pressure y in the chamber is approximately a linear function of the set point of flow (x). In this case, the quality characteristic y has a linear relationship with the independent variable x through $y = 2x + 1$, which must be monitored in Phase II when a shift of size 0.2 on the slope is taken place. Here, the aim is to implement an economic simple linear profile with good statistical properties. In other words, the lower limit for ARL_0 is considered 200 and a higher limit for ARL_1 is assumed 10. It is also assumed that the parameters of the Lorenzen-Vance cost function have been estimated as

$$E = 0.05, \gamma_1 = \gamma_2 = 1, T_0 = 0,\ T_1 = T_2 = 2\ , W = 25, \theta = 0.01, \tau = 0, F = 50, a = 0.5, b = 0.01$$

$\delta^2 = 0$, K= 0.1 P=200,

In addition, the parameters required to obtain the expected quality cost per unit in the Taguchi quadratic loss function when the process is in control (J_0) and when the process is out of control (J_1) are $P = 200, \delta = 0, T = 0, K = 0{,}1, \sigma_0^2 = 1, \mu_0 = 0,\ \mu_1 = 0.5, \sigma_1^2 = 1.5$. Here, K must be obtained using Eq. (20) using A as the cost of rework or scrap for each unit of the product. Then, using Eqs. (17) and (18) we estimate $J_0 = 0.05$ and $J_1 = 0.125$ by using Eq(17), and Eq(18). As a result, $C_0 = 200(0.05) = 10$ and $C_1 = 200(0.125) = 25$ using Eqs. (21) and (22), respectively.

The main parameters of the MOPSO algorithm as are described in the previous section are P_1 which is personal learning coefficient and P_2 which is the global learning coefficient and V (inertia weight). The other parameters of the MOPSO algorithm are $npop = nrep = 100$.

In the next sub-section, the parameters of MOPSO are tuned and the results are presented.

6.1. Determining the optimal parameters of the MOPSO algorithm

There are three main parameters involved in MOPSO which should be tuned before starting to solve a typical multi-objective optimization problem. These parameters are tuned in this section based on the steepest descent method described in Montgomery (2005). The main three parameters of MOPSO are the personal learning coefficient P_1, the global learning coefficient P_2, and the weight of the inertia V. For each of these parameters, a low, a medium, and a high level is first assumed. Next, the effect of each of these parameters on the implementation cost is analyzed using the analysis of variance (ANOVA) method applied on a factorial design. The response variable is considered the implementation cost obtained by the Lorenzen-Vance function. Table 2 shows the considered parameter levels of the MOPSO algorithm.

Table 2
The parameter levels of the MOPSO algorithm

Parameters	High	Medium	Low
P_1	2	1.5	1
P_2	2	1.5	1
V	2	1.5	1

Table 3 presents the responses obtained based on the parameter levels shown in Table 2 in a 2^3 factorial design, where -1 and 1 refer respectively to the low and the high level of a parameter.

Table 3
Responses in a factorial design of the economic-statistical model

Factor	*I*	*P₁*	*P₂*	*V*	*P₂*W*	*P₁*W*	*P₁*P2*	*P₁*P₂*V*	Response
I	1	-1	-1	-1	1	1	1	-1	12.15
P₁	1	1	-1	-1	1	-1	-1	1	12.45
P₂	1	-1	1	-1	-1	1	-1	1	12.40
V	1	1	1	-1	-1	-1	1	-1	12.33
*P₁*P₂*	1	-1	-1	1	-1	-1	1	1	12.06
*P₁*V*	1	1	-1	1	-1	1	-1	-1	11.99
*P₂*V*	1	-1	1	1	1	-1	-1	-1	11.80
*P₁*P₂*V*	1	1	1	1	1	1	1	1	11.68

From the above replicates, the average of the observed responses at factorial points is $\overline{y_F} = 12.11$. As there is one replicate at each factorial point, in order to estimate the variance of the error term as well as to investigate the curvature of the response function, 4 experiments are conducted at the center point, where the levels of all three parameters are chosen to be medium, i.e. at the $(1.5{,}1.5{,}1.5)$ point. The responses obtained at this point are 12.09, 12.34, 12.43, and 12.27 with an average of $\overline{y_C} = 12.28$. Consequently, the sum of squares of the pure quadratic (SSPQ) term is calculated using Eq. (26).

$$\mathrm{SSPQ} = \frac{n_c n_f (\overline{y_c} - \overline{y_f})^2}{n_c + n_f} = \frac{4 \times 8 \times (12.28 - 12.11)^2}{4 + 8} = 0.082 \quad (26)$$

The sum of squared error based on the results at the center point is $SSE = 0.2127$. This eventually results in the ANOVA table shown in Table 4.

Table 4
Analysis of variance on the parameters of the MOPSO algorithm

Source of variation	Sum of squares	DF	Mean of squares	F*
P_1	0.0002	1	0.0002	0.0033
P_2	0.0242	1	0.0242	0.3421
V	0.4013	1	0.4013	5.6594
P_1*P_2	0.0228	1	0.0228	0.3213
P_1*V	0.0220	1	0.0220	0.3105
P_2*V	0.0607	1	0.0607	0.8562
P_1*P_2*V	0.0120	1	0.0120	0.1689
PQ	0.0818	1	0.0818	1.1531
Error	0.2127	3	0.0709	

As $F_{0.1,1,3} = 5.54$, not only there is no curvature in the response function, but also the only significant parameter is V with an F-statistic equal to 5.6594. Therefore, the linear relationship is estimated in Eq. (27).

$$\hat{y} = 12.1088 - 0.22V \quad (27)$$

Employing the steepest descent method, Δ = (0,0,0.5) is chosen in each step to tune the parameters. Table 5 shows the results. Consequently, the tuned parameters of the MOPSO algorithm are $P_1 = 1, P_2 = 1,$ and $V = 3.5$.
The optimal parameters of the economic-statistical model, obtained by the application of the parameter-tuned MOPSO are shown in Table 6, where only the cost objective function, i.e. *ATL* is considered.

Table 5
Responses for different parameters along the regression line

Step	Parameters			Response
	P_1	P_2	V	
Origin	1	1	1	12.1089
Δ	0	0	0.5	
Origin+Δ	1	1	1.5	12.0410
Origin+2Δ	1	1	2	11.7712
Origin+3Δ	1	1	2.5	11.9438
Origin + 4Δ	1	1	3	11.8200
Origin+5Δ	1	1	3.5	11.8003
Origin+6Δ	1	1	4	11.8201

Table 6
The optimal solution generated by the MOPSO with tuned parameters

ATL	ARL_0	ARL_1	*ATS*	*n*	*r*	*h*	*L*
11.771	214.204	3.371	280.824	2	0.616	1.311	11.708

In the next subsection, an analysis is performed to compare the performance of the proposed method with the ones of some other competing works.

6.2. A comparison analysis

As is explained at the end of Section 2, this paper proposes a multi-objective optimization model for the economic-statistical design of linear profiles for the first time. All previous authors in the scope of economic-statistical design of control charts or linear profiles validated their models using an example. For instance, Chou and Cheng (2006), Noorossana et al. (2014) and Niaki & Ershadi (2011) validated their proposed optimization algorithms using an experimental design approach. In addition, Tavana et al. (2016), Niaki et al. (2012) and Barzinpour et al. (2013) compared their results to ones using a similar meta-heuristic solution algorithm. In this paper, in addition to using an experimental design for the validation of the proposed model and the solution algorithm, the results obtained by using the MOPSO is compared to the ones obtained by Niaki et al. (2011) using a PSO algorithm which solved a single-objective optimization problem. The comparison results are shown in Table 7. The utilization of the MOPSO algorithm of the current work leads to the total cost of 11.771, while the single- objective model after the application of the PSO results in the total cost of 15.63. This comparison shows that the multi-objective model achieves a better cost in comparison to the single-objective approach.

Table 7
The comparison of the single-objective to the multi-objective optimization model

Multi-objective model					Single-objective model				
Cost	*n*	*r*	*h*	*L*	*Cost*	*n*	*r*	*h*	*L*
11.771	2	0.616	1.311	11.708	15.63	7	0.91	1.52	13.88

In the next subsection, the efficiency of the proposed multi-objective optimization model is improved using DEA, when the parameter-tuned MOPSO is employed.

6.3. Efficiency improvement of the model

The efficiency is obtained by dividing the weighted composition of the outputs by the weighted composition of the inputs. Hence, to improve the efficiency, the weighted sum of the outputs must be maximized and the weighted sum of the inputs should be minimized. The inputs, the DMU, and the outputs involved in the economic-statistical design of simple linear profiles are shown in Figure 2.

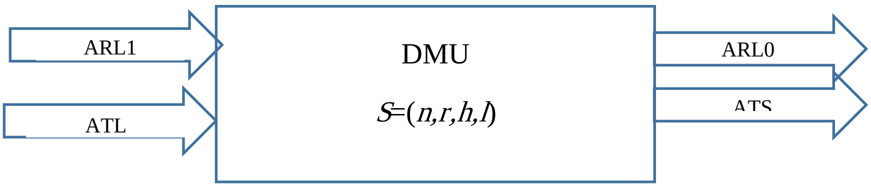

Fig. 2. Inputs and outputs for each DMU in the economic-statistical model

The inputs and the outputs of 43 DMUs are shown in Table 8 based on solving the multi-objective economic-statistical design using the parameter-tuned MOPSO. Each of these DMUs refers to a Pareto point obtained.

In order to obtain the efficiency of each DMU in Table 8, the method of conquering units is used by comparisons between the inputs and the outputs of the units. The outputs which are less than those outputs whose values are larger are recalled, and the inputs whose values are larger, are recalled by inputs with less values. Hence, efficient units are identified by eliminating some of the DMUs. The use of the method of conquering units by other units is appropriate for issues with low DMUs. Therefore, the linear programming model is used here to obtain the optimal efficiency of each DMU. As an example, the linear programming model of the first DMU is as follows

$Max\ Z_1=237.6339u_1+1325.664u_2$

$s.t.$

$13.1259v_1 + 2.804v_2 \leq 1$

$237.6339u_1+1325.664u_2-13.1259v_1-2.804v_2 \leq 0$

$243.7553u_1+1631.064u_2-13.002v_1-2.835v_2 \leq 0$

.

$229.6277u_1+767.752u_2-12.4487v_1-3.8497v_2 \leq 0$

$u_1, u_2, v_1, v_2 \geq 0$

To obtain the efficiencies of the other 42 DMUs, 42 such models that are different in the target function and the first limitation are to be solved. The optimal efficiencies of all DMUs are presented in Table 9.

Table 8
The inputs and outputs of 43 DMUs

Unit	Outputs		Inputs		Unit	Outputs		Inputs	
	ATS	ARL_0	ARL_1	*ATL*		*ATS*	ARL_0	ARL_1	*ATL*

1	1325.664	237.634	2.804	13.126	23	1331.105	245.972	2.904	12.770
2	1631.064	243.755	2.835	13.002	24	746.819	228.490	3.623	12.521
3	2042.049	228.614	2.812	13.741	25	1336.026	229.560	2.852	12.870
4	1368.003	227.564	2.938	12.949	26	534.393	247.886	7.020	12.912
5	694.928	237.217	3.420	12.476	27	2428.283	244.528	3.907	14.683
6	987.083	236.291	3.347	12.587	28	1633.479	248.816	2.852	13.161
7	2178.441	242.049	3.028	13.818	29	2323.727	232.373	3.313	14.264
8	2293.737	247.143	3.2871	14.164	30	1070.846	249.231	4.851	13.336
9	1068.682	206.198	2.806	12.68	31	829.428	206.603	2.924	12.350
10	280.822	214.204	3.371	11.771	32	1869.783	236.176	2.901	13.437
11	2045.308	249.949	5.749	15.369	33	1849.473	249.861	2.990	13.442
12	906.878	212.598	2.960	12.464	34	1233.799	249.944	5.070	13.716
13	2163.596	248.929	5.885	15.633	35	2488.093	248.809	4.269	15.027
14	2015.993	249.934	3.178	13.827	36	1549.004	243.137	2.946	12.968
15	859.051	214.763	2.844	12.646	37	1523.795	247.598	2.999	13.025
16	1060.959	233.872	2.897	12.617	38	2236.396	247.377	3.935	14.559
17	2220.276	239.890	2.872	13.847	39	2142.553	249.761	5.926	15.601
18	1731.653	244.805	3.083	13.422	40	1640.752	249.131	2.882	14.286
19	2248.453	249.828	5.762	15.670	41	767.255	229.628	3.850	12.449
20	2141.567	245.294	2.855	13.665	42	2221.860	247.832	3.040	13.850
21	834.477	220.307	2.939	12.519	43	2010.943	203.675	2.802	13.856
22	2260.614	245.223	3.123	13.992					

Table 9
The efficiencies of DMUs

Unit	Z^*	Unit	Z^*
1	0.43	23	1.00
2	0.38	24	0.67
3	0.56	25	0.91
4	1.00	26	0.78
5	0.83	27	0.54
6	0.47	28	0.76
7	0.65	29	0.75
8	0.46	30	0.81
9	1.00	31	0.57
10	0.57	32	0.81
11	0.85	33	0.91
12	0.61	34	1.00
13	0.39	35	0.65
14	0.59	36	0.74
15	0.87	37	0.67
16	0.54	38	0.89
17	0.39	39	1.00
18	0.93	40	0.38
19	0.82	41	0.75
20	1.00	42	0.59
21	0.71	43	0.47
22	0.81		

In Table 10, the DMUs whose *Z*-values are 1.00 are effective units and the ones whose *Z*-values are greater than or equal to 0.9 are relatively efficient. The units with *Z*-values less than 0.9 are inefficient units. Table 10 summarizes the inputs and the outputs of the efficient and relatively efficient DMUs.

In the next section, some sensitivity analyses are performed on the main parameters of the proposed economic-statistical model.

Table 10
The inputs and the output variables of the efficient and relatively efficient DMUs

Unit	*n*	*R*	*h*	*L*	*ATL*	*ARL1*	*ARL0*	*ATS*	efficiency	
4	6	0.552	6.012	9.383	12.949	2.938	227.564	1368.003	1.00	Efficient
9	7	0.818	5.183	13.681	12.680	2.806	206.198	1068.682	1.00	Efficient
20	11	0.417	8.731	13.401	13.665	2.855	245.294	2141.567	1.00	Efficient
23	6	0.484	5.412	11.152	12.770	2.904	245.972	1331.105	1.00	Efficient
34	7	0.571	4.936	13.723	13.716	5.070	249.944	1233.799	1.00	Efficient
39	11	0.723	8.578	15.000	15.601	5.926	249.761	2142.553	1.00	Efficient
25	7	0.635	5.820	10.614	12.870	2.852	229.558	1336.026	0.93	Relatively efficient
33	11	0.570	7.402	12.936	13.442	2.990	249.861	1849.473	0.91	Relatively efficient

6.3. Sensitivity analyses

As many parameters of the Lorenzen-Vance cost need to be estimated in order to design a proper multi-objective economic-statistical design of simple linear profiles, in this section the effects of under-estimation and over-estimation of some of the parameters on the design are investigated in some sensitivity analyses. Tables 11-14 show the optimal designs when the fixed cost of sampling (a), the variable cost of sampling (b), the plotting cost of each profile (e), and the cost of identifying and modifying an assignable cause (W) are changed.

Table 11
The effect of the fixed cost of sampling on the design

	ATL	ARL_1	ARL_0	*ATS*	*n*	*r*	*h*	*l*
a=5	13.437	2.836	248.644	1106.503	4	0.642	4.450	8.889
a=0.05	11.280	3.089	202.232	216.651	2	0.242	1.071	10.351
a=0.5	12.445	2.895	238.060	950.791	2	0.580	3.994	10.498

Table 12
The effect of the variable cost of sampling on the design

	ATL	ARL_1	ARL_0	*ATS*	*n*	*r*	*h*	*l*
b=0.1	11.267	3.118	247.441	325.678	13	0.427	1.316	12.000
b=0.01	12.088	2.884	237.678	791.874	9	0.373	3.332	8.864
b=0.001	11.593	2.840	232.300	347.035	8	0.459	1.494	9.309

Table 13
The effect of the plotting cost of each profile on the design

	ATL	ARL_1	ARL_0	*ATS*	*n*	*r*	*h*	*l*
e=0.05	12.712	2.940	220.634	1066.030	9	0.590	4.831	10.248
e=0.5	13.090	3.600	230.280	739.620	10	0.563	3.212	12.451
e=5	160.762	2.961	235.509	1375.030	2	0.629	5.838	10.862

Table14
The effect of *W* on the design

	ATL	*ARL1*	*ARL0*	*ATS*	*n*	*r*	*h*	*l*
W=20	11.692	2.811	231.056	386.586	2	0.271	1.673	11.804
W=150	13.943	4.108	238.008	298.652	12	0.120	1.255	10.019
W=250	13.274	2.925	244.199	754.459	8	0.312	3.089	10.967

The results in Tables 11-14 show that while the proposed model is robust on some parameters such as the cost of identifying and modifying an assignable cause, the fixed cost of each sample, and the variable cost of sampling, the plotting cost of each profile has a significant effect on the design. Hence, the estimation of this parameter is an important task.

7. Conclusions and Recommendations for Future Research

In this paper, a multi-objective economic-statistical design of simple linear profiles was proposed. This means that the design parameters of a profile can be obtained in a way the total implementation cost is minimized while desired statistical properties are achieved. The objectives of the problem included minimizing the implementation cost, maximizing the average run length when the process is in control, maximizing the average time to signal, and minimizing the average run length when the process is out of control. While an EWMA-R scheme was employed for statistical monitoring of the profile, the Lorenzen-Vance cost function was used to consider hidden implementation costs estimated by the Taguchi loss function. A MOPSO algorithm, for which its parameters were tuned using RSM was utilized to solve the complex multi-objective optimization problem. In addition, the concept of DEA was used to obtain the optimal effective solutions generated by the MOPSO. Finally, some sensitivity analyses were conducted on the main parameters of the Lorenzen-Vance cost function. The results showed that while the design demonstrated a robust performance on some parameters, the plotting cost of each profile had a significant effect on the design. This implied that care must be taken in order to estimate this parameter.
In the proposed model, only one type of assignable cause was assumed. A model that can accommodate several types of assignable causes can be considered in the future. In addition, the model can be extended to design other types of profiles such as multivariate and non-linear profiles. The use of quality function deployment is also recommended in the future in order to estimate the costs. Moreover, some other decision-making methods can be used to rank the Pareto optimal solutions. Finally, adaptive modes can be added to the designed model.

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